

**ECHO CANCELLATION**

**Y562 FINAL PROJECT**

**Kenneth A. Cobb  
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Prof. S. B. Kesler**

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### REFERENCES

## 1.0 INTRODUCTION

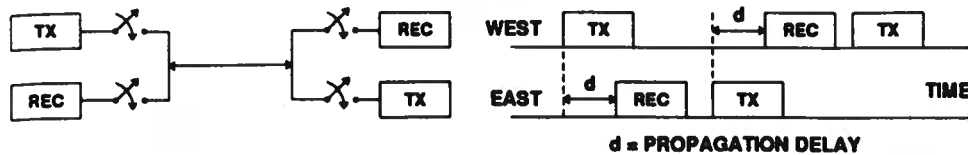
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Multiple access communications relies on the orthogonality of different signals by separating them in time or frequency. Several techniques are applied to accomplish full-duplex transmission of signals, or simultaneous transmission in both directions, on a point-to-point link. Two such techniques are Time-Compression Multiplexing (TCM) and frequency-division multiplexing (FDM). Both of these techniques require separate transmit and receive frequency bands. A more efficient technique, called Echo Cancellation, enables transmission in both directions simultaneously using the same frequency band, thus reducing the bandwidth requirements approximately in half, relative to TCM and FDM.

This term paper will attempt to provide a brief background of TCM and FDM, and describe the concept of echo cancellation and its application to Baseband and Passband signals. Finally, this paper will describe the two measures of performance of an adaptive echo canceler, and the existing tradeoffs between these two measures.

## 2.0 BACKGROUND

Time-compression multiplexing, one of the methods mentioned earlier, provides full-duplex transmission. "Time-compression" refers to the fact that a bit-stream in one direction is divided into traffic bursts and transmitted at a speed at least twice as high as its average bit rate. TCM is illustrated in the figure below:

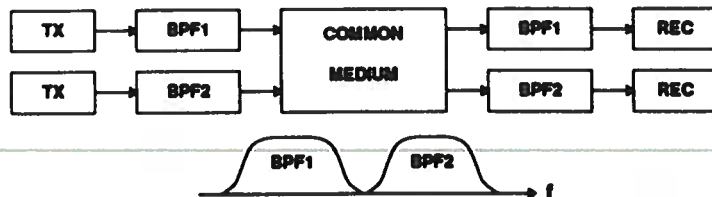


The switch closes to connect a local transmitter to a remote receiver, alternating in direction. The "west" transmitter transmits a traffic burst at regular intervals (to insure the fixed bit rate) and at a rate somewhat higher than twice the average data rate in one direction. The west transmitter is the master for the system, defining the basic time-division multiple access frame. It does not transmit a reference burst, but rather its traffic burst, and the preamble serves to define the frame. The combined reference and traffic burst from the "west" end is received at the "east"  $d$  seconds later, where  $d$  is the propagation delay on the medium. The "east" end, which is slaved to the "west" end, detects this received traffic burst and sends its own traffic burst shortly thereafter. This burst arrives at the "west" end with delay  $d$ , hopefully finishing before the end of the frame.

The choice of an instantaneous data rate, the length of a burst, and a

desired guard time between bursts puts an upper limit on  $d$  and therefore, the distance of transmission. Arbitrarily large delays will result in overlap of the received and transmitted traffic bursts at the "west" end. Because of the limitation on distance, time-compression multiplexing is only useful on telephone lines, and is not efficient for voiceband data full-duplex transmission because of the possibility of very long delays on satellite connections.

Alternatively, frequency-division multiplexing is used to separate many users on a common medium in analog transmission. FDM is illustrated in figure below:



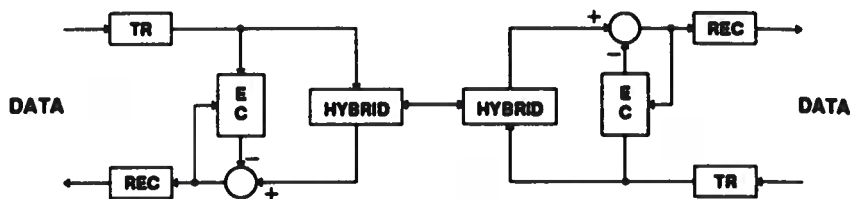
The two transmitters sharing the medium have output power spectra in two non-overlapping bands, where they usually use passband PAM demodulation to achieve this. To ensure this, it is common to put bandpass filters at the output of the transmitters. At the two receivers, similar bandpass filters eliminate all but the desired data signals. The path to a receiver from the undesired transmitter contains two bandpass filters with non-overlapping passbands. Hence, the loss of the crosstalk path can be made as large as desired through filter design.

FDM has both advantages and disadvantages over TDM. A major disadvantage on all media is the relatively expensive and complicated bandpass filters required, whereas, TDM is realized primarily with much cheaper logic functions. Another disadvantage of FDM is the rather strict linearity requirement of the medium. However, some of the propagation-delay and crosstalk problems of TDM are eliminated by FDM.

As was mentioned earlier, echo cancellation uses the same frequency band for transmission in both directions, and is thereby more efficient. The principle of echo cancellation is that at each end of a full-duplex link, the near-end transmitted signal can be used to eliminate the undesired interference, called an echo, of the near-end transmitted signal at the receiver. The echo canceler can learn adaptively the response from near-end transmitter to receiver, generate a replica of that echo, and subtract that echo replica from the receiver input to yield an interference-free signal.

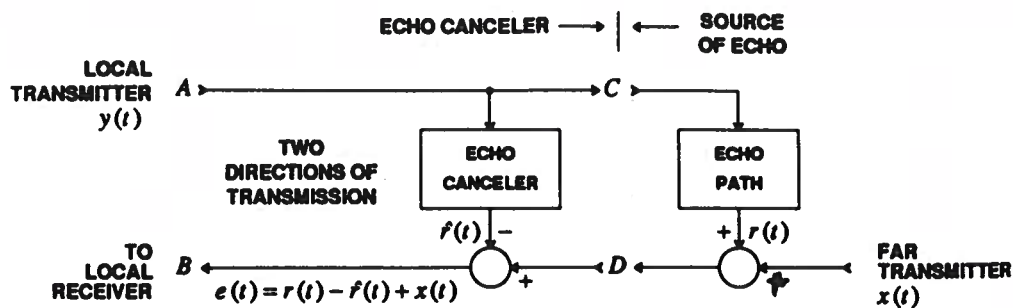
### 3.0 ECHO CANCELER

When a data signal is transmitted full-duplex from a transmitter to a receiver, the primary problem is undesired feed-through of the transmitted signal through what is called the hybrid. The hybrid separates the two directions of transmissions when both signals share the same wire pair. This undesired feed-through is called the echo. The mechanism for echo is a mismatch between the impedance of the two-wire cable and the hybrid balancing impedance. The echo cancellation method of transmission is shown in the figure below:



There is a transmitter and receiver on each end of the connection, and the hybrid is used to provide a virtual four-wire connection between the transmitter on each end and the receiver on the opposite end. The echo canceler is an adaptive transversal filter that learns the response of the hybrid, and generates a replica of that response which is subtracted from the hybrid output to yield an echo-free received signal.

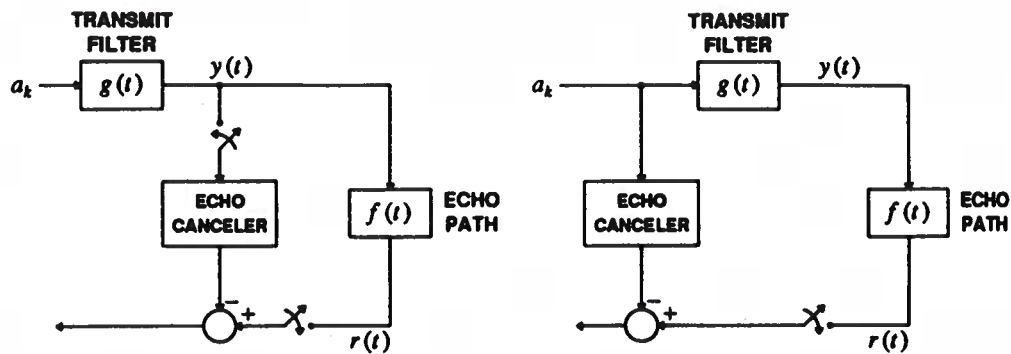
Echo canceler notation is shown in the figure at the top of the next page:



The local transmitter signal  $y(t)$  at port A generates the undesired echo signal  $r(t)$ . This signal is superimposed at the output of the hybrid, port D, with the far transmitter signal  $x(t)$ . The canceler takes advantage of its knowledge of the local transmitter signal to generate a replica of the echo,  $\hat{r}(t)$ . This replica is subtracted from the echo plus far transmitter signal  $x(t)$  alone. The echo canceler is usually implemented in discrete-time as a finite transversal filter. The canceler design depends strongly on the details of the local transmitter and receiver design.

Two common echo cancelers are the Baseband channel echo canceler and the more complicated Passband channel echo canceler. Two configurations for a baseband channel echo canceler are shown in the figure at the top of the next page, where  $a_k$  is the sequence of transmitted data symbols,  $g(t)$  is the transmitted pulse shape,  $y(t)$  is the transmitted data waveform, assumed to be real-valued, and  $r(t)$  is the echo response:





In the left figure, the transmitted data waveform  $y(t)$  is sampled at the canceler input, while in right figure, the transmitted data symbols are applied directly to the canceler input so that the transmit filter is included in the echo path.

Because the transmitted and echo signals have bandwidth greater than half the baud rate, a sampling rate of twice the baud rate or more is required in left figure above. On the other hand, the sampling rate at the input of the canceler in the right figure is equal to the baud rate, which leads to the conclusion that the sampling rate at the output of the echo canceler is higher than the sampling rate at the input.

A possible solution to the problem of incompatible sampling rate is called Interleaved Echo Cancelers. The Interleaved Echo Canceler takes advantage of the availability of a clock representing the transmitted data signal, and samples the echo signal at a rate that is an integer

multiple of the transmit baud rate, for example a multiple of R:

$$r_1(l) = r((i + l/R)T), \quad 0 \leq l \leq R-1,$$

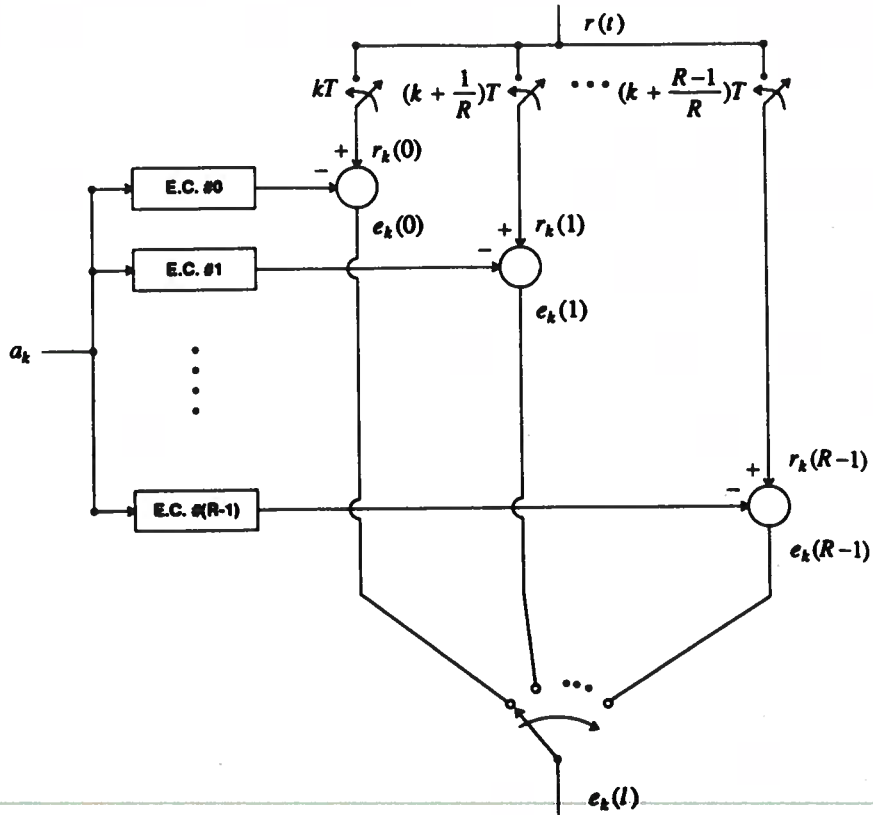
where the index  $i$  represents the data symbol event and  $l$  represents the sample from among  $R$  samples uniformly spaced in this event. Similarly, define a notation for the samples of the echo pulse response:

$$h_1(l) = h((i + l/R)T), \quad 0 \leq l \leq R-1,$$

Therefore,  $r_1(l) = \sum_{m=-\infty}^{\infty} h_m(l) a_{i-m}$ , which shows that the sample of the echo can be thought of as  $R$  independent echo channels, each being driven by an identical sequence of data symbols.

The echo replica can be generated independently for each echo channel by a series of  $R$  interleaved echo cancelers, as shown in figure at the top of the next page. Each canceler cancels the echo for one sampling phase, from among  $R$ , and has a sampling rate at the input and output equal to the baud rate. Each canceler operates independently of the other. Each canceler can be thought of as adapting to the impulse response of the echo channel sampled at a rate equal to the baud rate, but with a particular phase out of  $R$  possible phases. Since the transversal filters all adapt independently, the presence of multiple interleaved canceler filters does not affect the speed of adaptation.

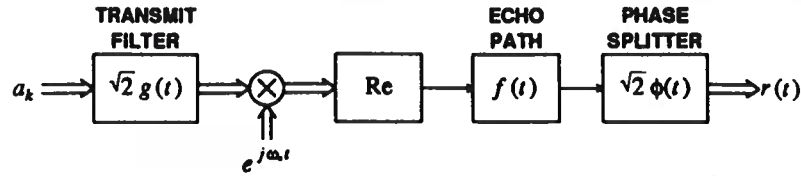
Examining the figure on page 7 again, the interleaved canceler required in right figure has important advantages over the left figure:



- The input to the canceler is transmitted data symbols, with a finite (usually small) alphabet. The implementation of the canceler therefore requires a relatively simple multiplier, since the transmitted data symbols have a very few bits of precision.
- The speed of adaptation is greater, since the interleaved cancelers adapt independently and each has fewer taps.
- The canceler complexity as measured by the multiplication rate is lower.

The Passband Echo Canceler is considerably different since: 1) the canceler input is complex-valued, and 2) the transmit modulator is included in the transmit path, so that the echo path is time-varying. It is assumed that the carrier frequency and phase are precisely

known. A model for the transmitter and echo path is shown in the figure below:



where  $g(t)$  is the transmit filter and  $f(t)$  is the echo path impulse, and the received echo signal is:

$$r(t) = 2\text{Re}\left\{ \sum_{k=-\infty}^{\infty} a_k h(t - kT) e^{j\omega_c t} \right\} ,$$

After a minor manipulation, this can be written in the form:

$$r(t) = \sum_k \tilde{a}_k \tilde{h}(t - kT) ,$$

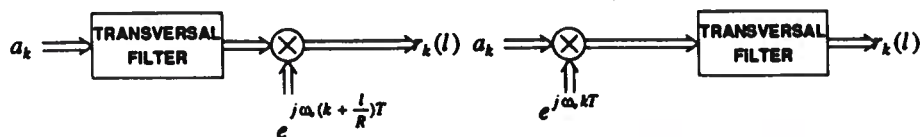
where  $\tilde{h}(t) = h(t)e^{j\omega_c t}$  is an equivalent passband pulse waveform, and  $\tilde{a}_k = a_k e^{j\omega_c kT}$  is called the rotated data symbol since it is rotated by the angle  $\omega_c kT$  radians.

As in the baseband case, the sampling rate at the receiver input will generally be a multiple  $R$  of the baud rate, necessitating interleaved echo cancelers. Therefore, the echo channel model can also be written:

$$r(t) = \left[ \sum_m a_m h_{k-n}(l) \right] e^{j\omega_c (k+1/R)T}$$

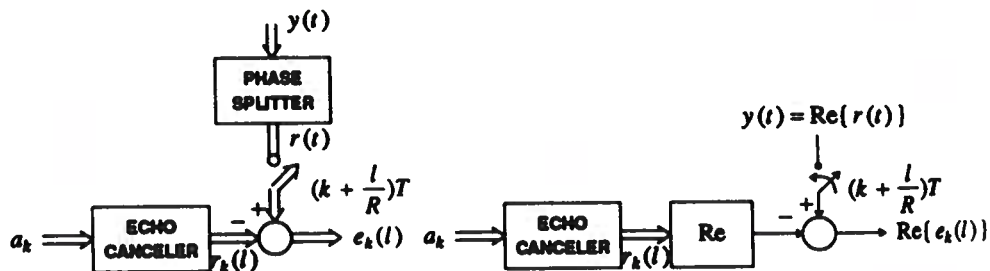
Comparing the passband and the baseband transversal filters, the main difference is the placement of the modulator after or before the complex-coefficient transversal filter. The figure below shows two

configurations for one interleaved echo canceler corresponding to a passband channel:



The left figure is a baseband transversal filter followed by a modulator, while the right figure is a modulator followed by a passband transversal filter.

Another option in cancelers for a passband channel is the generation of a real-valued or complex-valued error signal as in the figure below:



The figure on the left is the one considered so far. It is assumed that the receive analytic signal is generated using a phase splitter, and the echo canceler generates a replica of the echo analytic signal.

The real-error alternative, on the right, cancels only the real part of the analytic signal, which in actuality is the passband receive

waveform. For this case, only the real part of the canceler complex-valued output is required. Use of the real-error canceler can reduce the canceler computational load because only the real part of the output need be calculated. Similarly, the receive signal is used in place of the analytic signal, the receive signal being the real part of the analytic signal, thus eliminating the need for the phase splitter. Overall, the complexity of the real-error canceler is lower. However, the convergence of the complex-error canceler is faster than that of the real-error canceler, because the complex-error canceler makes use of more information. Further, in some instances, the savings of a phase splitter in a real-error canceler is negated by the need for a splitter in the data receiver that follows the echo canceler.

#### 4.0 ADAPTATION

There are two measures of performance of an adaptive echo canceler: 1) the speed of adaptation, and 2) the accuracy of the cancellation after adaptation. There is a tradeoff between these two measures. For a particular class of adaptation algorithm, as the speed of adaptation is increased, the accuracy of the transfer function after adaptation gets poorer. This tradeoff is fundamental, since a longer averaging time is necessary to increase asymptotic accuracy, but slows the rate of convergence. However, in most instances, the accuracy of the final cancellation of the echo is the most critical design factor.

In addition, while the ability of the canceler to rapidly track a changing echo response is usually not important, the speed of initial adaptation from an arbitrary initial condition is often important.

The goal is to calculate the optimum tap coefficients for a complex-error passband transversal filter canceler. These tap coefficients can be defined by a vector of  $N$  filter coefficients:

$$c' = [c_0, c_1, c_2, \dots, c_{N-1}]$$

For the passband transversal filter canceler, the minimum mean-square error solution and the optimum tap coefficients are derived in Reference 1, and are found to be:

$$c_{opt} = \tilde{h} + \Phi^{-1}p, \quad \xi_{min} = \sigma_v^2 - p^* \Phi^{-1}p,$$

where  $\tilde{h}$  is a vector of the first  $N$  impulse response samples,  $\Phi$  is the autocorrelation matrix of the vector of the current and  $N-1$  past input rotated data symbols  $\tilde{a}_k$ ,  $p$  is the correlation of the residual

uncancelable echo and vector of the current and N-1 past input rotated data symbols  $\tilde{a}_k$ , and  $\sigma_v^2$  is the variance of the uncancelable echo.

The most widely used adaptation algorithm for the echo canceler is the stochastic gradient (SG) algorithm. For the passband transversal filter case, the first step is to determine the magnitude-squared of the analytic cancellation error as a function of the tap coefficient vector  $c$ , and then take the gradient of this expression with respect to  $c$ . The SG algorithm follows from evaluating this gradient at the last coefficient vector, multiplying by step size  $\beta/2$ , and subtracting the result from the last coefficient vector to get a new coefficient vector:

$$c_k = c_{k-1} + \beta E_k \tilde{a}_k^* \quad , \quad E_k = R_k - c_{k-1}^H \tilde{a}_k \quad .$$

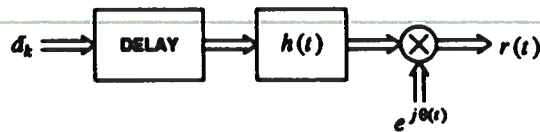
When examining the convergence of the real-error canceler versus the complex-error canceler, for the same step-size  $\beta/2$ , the real-error canceler converges with a time constant that is approximately twice as great as the complex-error canceler. This makes sense because the real-error canceler is disposing of half of the information available. Both cancelers have approximately the same asymptotic mean-square error.



## 5.0 FAR-END ECHO

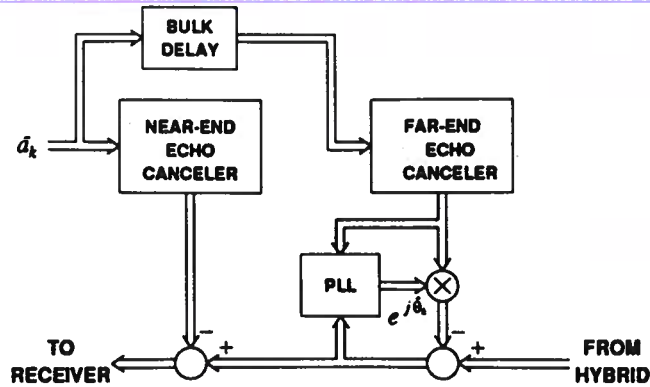
When a data signal is transmitted over long distances via telephone modem, echo can occur not only at the near-end in conjunction with the four-wire to two-wire converter, but also at intermediate points in the telephone network. This type of echo is called Far-end echo. Far-end echos are generally more attenuated than the near-end echo, and require a less accurate cancellation. However, they are subject to additional impairments such as jitter and frequency offset. Therefore, very accurate cancellation of these echos requires the addition of algorithms to the basic echo canceller considered in this paper.

One such model is shown in the figure below, and is similar to the passband filter operating on the rotated symbols:



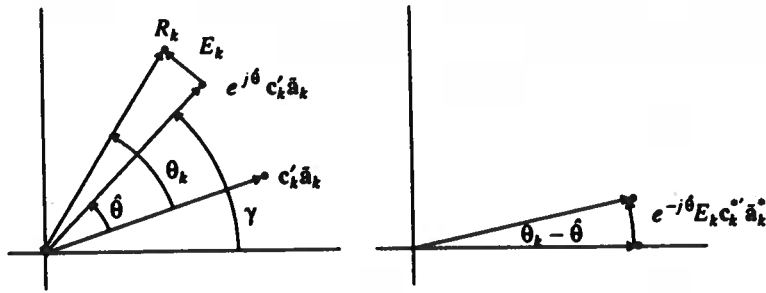
Added are two features: 1) A bulk delay accounting for the propagation delay from the transmitter to the point of echo generation, and 2) a carrier phase rotation by angle  $\theta(t)$  at the output to account for possible phase jitter and frequency offset in the echo channel.

The figure on the top of the next page shows the configuration for a voiceband data modem canceler which utilizes the far-end echo mechanism. This passband echo canceler consists of a bulk delay, which hopefully matches the delay of the echo channel, a passband



transversal filter, and a phase rotator by angle  $\hat{\theta}_k$  which hopefully matches the carrier phase rotation  $\theta_k$  of the echo channel. The appropriate angle for rotation is determined by a phase-locked loop, which uses the transversal filter output and cancellation error to correct the currently used phase in order to minimize the error signal  $E_k$ .

A phased-locked loop algorithm can be derived using a stochastic gradient (SG) approach to take the derivative of  $|E_k|^2$ , the mean-square error, with respect to the PLL output phase  $\hat{\theta}_k$ , where  $E_k$  is the difference between the echo signal and echo replica. By adjusting  $\hat{\theta}_k$  in the opposite direction of this derivative, the phase error  $\theta_k - \hat{\theta}_k$  can be tracked. The graphical interpretation is shown in left figure on the top of the next page.  $\theta$  is the angle of the echo replica relative to the real-axis. The goal is for the echo replica  $e^{j\theta} c_k \tilde{a}_k$  to equal the echo signal  $R_k$ , or to have the error signal  $E_k$  equal to zero.



In the right figure above, the entire orientation is rotated by  $-\delta$  such that the echo replica,  $e^{j\delta} c_k' \tilde{a}_k$ , appears on the real-axis. This way, it becomes easy to determine whether the phase error  $\theta_k - \hat{\theta}$  is positive or negative. If this imaginary part is positive, then the error is reduced by making the estimated phase  $\hat{\theta}$  larger.

## REFERENCES

1. Digital Communication, by E. A. Lee and D. G. Messerschmitt, 1988.
2. Echo Cancellation in Speech and Data Transmission, by D. G. Messerschmitt, Advanced Digital Communications Systems and Signal Techniques, 1987.
3. Data and Computer Communications, by W. Stallings, 1985.